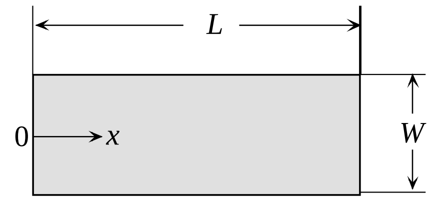
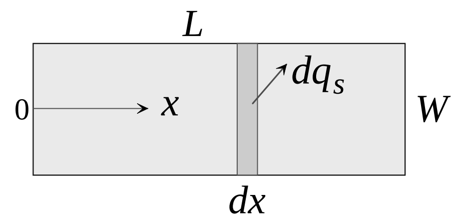
PROBLEM 1.1

Heat is removed from a rectangular surface by convection to an ambient fluid at T_∞. The heat transfer coefficient is h. Surface temperature is given by

$$T_s = \frac{A}{x^{1/2}}$$



where A is constant. Determine the steady state heat transfer rate from the plate.



(1) Observations. (i) Heat is removed from the surface by convection. Therefore, Newton's law of cooling is applicable. (ii) Ambient temperature and heat transfer coefficient are uniform. (iii) Surface temperature varies along the rectangle.

(2) Problem Definition. Find the total heat transfer rate by convection from the surface of a plate with a variable surface area and heat transfer coefficient.

(3) Solution Plan. Newton's law of cooling gives the rate of heat transfer by convection. However, in this problem surface temperature is not uniform. This means that the rate of heat transfer varies along the surface. Thus, Newton’s law should be applied to an infinitesimal area dA_s and integrated over the entire surface to obtain the total heat transfer.

(4) Plan Execution.

(i) Assumptions. (1) Steady state, (2) negligible radiation, (3) uniform heat transfer coefficient and (4) uniform ambient fluid temperature.

(ii) Analysis. Newton's law of cooling states that

$$q_s = h\, A_s\, (T_s - T_\infty) \tag{a}$$

where

A_s = surface area, m^2
h = heat transfer coefficient, $W/m^2\text{-}^oC$
q_s = rate of surface heat transfer by convection, W
T_s = surface temperature, ^oC
T_∞ = ambient temperature, ^oC

Applying (a) to an infinitesimal area dA_s

$$dq_s = h\,(T_s - T_\infty)\, dA_s \tag{b}$$

The next step is to express $T_s(x)$ in terms of distance x along the triangle. $T_s(x)$ is specified as

$$T_s = \frac{A}{x^{1/2}} \tag{c}$$

The infinitesimal area dA_s is given by

$$dA_s = W\,dx \tag{d}$$

where

x = axial distance, m
W = width, m

Substituting (c) and into (b)

$$d\,q_s = h(\frac{A}{x^{1/2}} - T_\infty)\,Wdx \tag{e}$$

Integration of (f) gives q_s

$$q_s = \int dq_s = hW\int_0^L (Ax^{-1/2} - T_\infty)dx \tag{f}$$

Evaluating the integral in (f)

$$q_s = hW\left[2AL^{1/2} - LT_\infty\right]$$

Rewrite the above

$$q_s = hWL\left[2AL^{-1/2} - T_\infty\right] \tag{g}$$

Note that at $x = L$ surface temperature $T_s(L)$ is given by (c) as

$$T_s(L) = AL^{-1/2} \tag{h}$$

(h) into (g)

$$q_s = hWL\left[2T_s(L) - T_\infty\right] \tag{i}$$

(iii) Checking. *Dimensional check*: According to (c) units of C are $^{o}\text{C/m}^{1/2}$. Therefore units q_s in (g) are W.

Limiting checks: If $h = 0$ then $q_s = 0$. Similarly, if $W = 0$ or $L = 0$ then $q_s = 0$. Equation (i) satisfies these limiting cases.

(5) Comments. Integration is necessary because surface temperature is variable.. The same procedure can be followed if the ambient temperature or heat transfer coefficient is non-uniform.

PROBLEM 1.2

A right angle triangle is at a uniform surface temperature T_s. Heat is removed by convection to an ambient fluid at T_∞. The heat transfer coefficient h varies along the surface according to

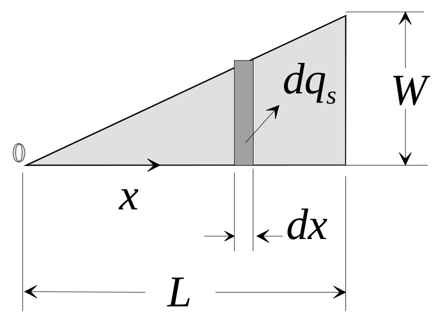
$$h = \frac{C}{x^{1/2}}$$

where C is constant and x is distance along the base measured from the apex. Determine the total heat transfer rate from the triangle.

(1) Observations. (i) Heat is removed from the surface by convection. Therefore, Newton's law of cooling may be helpful. (ii) Ambient temperature and surface temperature are uniform. (iii) Surface area and heat transfer coefficient vary along the triangle.

(2) Problem Definition. Find the total heat transfer rate by convection from the surface of a plate with a variable surface area and heat transfer coefficient.

(3) Solution Plan. Newton's law of cooling gives the rate of heat transfer by convection. However, in this problem surface area and heat transfer coefficient are not uniform. This means that the rate of heat transfer varies along the surface. Thus, Newton's law should be applied to an infinitesimal area dA_s and integrated over the entire surface to obtain the total heat transfer.



(4) Plan Execution.

(i) Assumptions. (1) Steady state, (2) negligible radiation and (3) uniform ambient fluid temperature.

(ii) Analysis. Newton's law of cooling states that

$$q_s = h\,A_s\,(T_s - T_\infty) \tag{a}$$

where

A_s = surface area, m^2
h = heat transfer coefficient, $W/m^2\text{-}^oC$
q_s = rate of surface heat transfer by convection, W
T_s = surface temperature, ^oC
T_∞ = ambient temperature, ^oC

Applying (a) to an infinitesimal area dA_s

$$dq_s = h\,(T_s - T_\infty)\,dA_s \tag{b}$$

The next step is to express h and dA_s in terms of distance x along the triangle. The heat transfer coefficient h is given by

$$h = \frac{C}{x^{1/2}} \tag{c}$$

The infinitesimal area dA_s is given by

$$dA_s = y(x)\,dx \tag{d}$$

where

x = distance along base of triangle, m
$y(x)$ = height of the element dA_s, m

Similarity of triangles give

$$y(x) = \frac{W}{L}x \tag{e}$$

where

L = base of triangle, m
W = height of triangle, m

Substituting (c), (d) and (e) into (b)

$$dq_s = \frac{C}{x^{1/2}}(T_s - T_\infty)\,\frac{W}{L}x\,dx \tag{f}$$

Integration of (f) gives q_s. Keeping in mind that C, L, W, T_s and T_∞ are constants, (f) gives

$$q_s = \int dq_s = \frac{CW}{L}(T_s - T_\infty)\int_0^L \frac{x}{x^{1/2}}\,dx \tag{g}$$

Evaluating the integral in (g)

$$q_s = \frac{2}{3}C\,W\,L^{1/2}\,(T_s - T_\infty) \tag{h}$$

(iii) Checking. *Dimensional check*: According to (c) units of C are W/m$^{3/2}$-^{o}C. Therefore units of q_s in (h) are

$$q_s = C(\text{W/m}^{3/2}\text{-}^o\text{C})\ W(\text{m})\ L^{1/2}(\text{m}^{1/2})\ (T_s - T_\infty)(^o\text{C}) = \text{W}$$

Limiting checks: If $h = 0$ (that is $C = 0$) then $q_s = 0$. Similarly, if $W = 0$ or $L = 0$ or $T_s = T_\infty$ then $q_s = 0$. Equation (h) satisfies these limiting cases.

(5) Comments. Integration was necessary because both area and heat transfer coefficient vary with distance along the triangle. The same procedure can be followed if the ambient temperature or surface temperature is non-uniform.

PROBLEM 1.3

A high intensity light bulb with surface heat flux $(q/A)_s$ *is cooled by a fluid at* T_∞. *Sketch the fluid temperature profiles for three values of the heat transfer coefficients:* h_1, h_2, *and* h_3, *where* $h_1 < h_2 < h_3$.

(1) Observations. (i) Heat flux leaving the surface is specified (fixed). (ii) Heat loss from the surface is by convection and radiation. (iii) Convection is described by Newton's law of cooling. (iv). Changing the heat transfer coefficient affects temperature distribution. (v). Surface temperature decreases as the heat transfer coefficient is increased. (vi) Surface temperature gradient is described by Fourier's law.(vii) Ambient temperature is constant.

(2) Problem Definition. Determine effect of heat transfer coefficient on surface temperature and surface gradient..

(3) Solution Plan. (i) Apply Newton's law of cooling to examine surface temperature. (ii) Apply Fourier's law to determine temperature gradient at the surface.

(4) Plan Execution.

(i) Assumptions. (1) Steady state, (2) no radiation ,(3) uniform ambient fluid temperature and (4) constant properties.

(ii) Analysis. Newton's law of cooling

$$(q/A)_s = h(T_s - T_\infty) \tag{a}$$

Solve for T_s

$$T_s = T_\infty + \frac{(q/A)_s}{h} \tag{b}$$

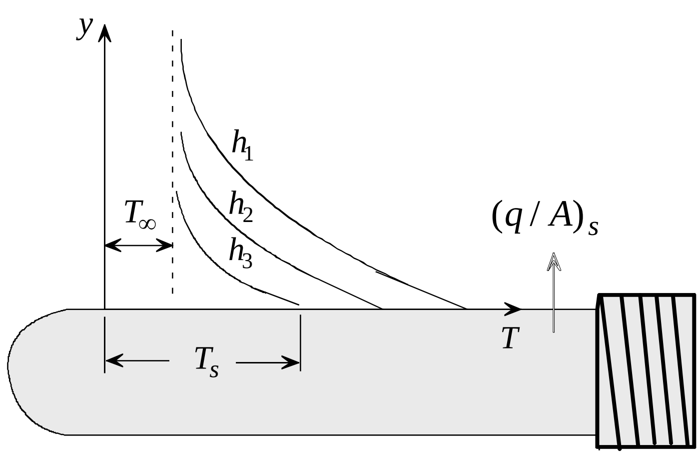
This result shows that for constant $(q/A)_s$ surface temperature decreases as h is increased. Apply Fourier's law

$$(q/A)_s = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{c}$$

where y is the distance normal to the surface. Rewrite (c)

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\frac{(q/A)_w}{k} \tag{d}$$

This shows that temperature gradient at the surface remains constant independent of h. Based on (b) and (d) the temperature profiles corresponding to three values of h are shown in the sketch.



(iii) Checking. *Dimensional check*: (1) Each term in (b) has units of temperature

$$T_s(^\circ\text{C}) = T_\infty(^\circ\text{C}) + \frac{(q/A)_s(\text{w/m}^2)}{h(\text{w/m}^2 - {}^\circ\text{C})} = {}^\circ\text{C}$$

(2) Each term in (d) has units of $^{\text{o}}\text{C/m}$

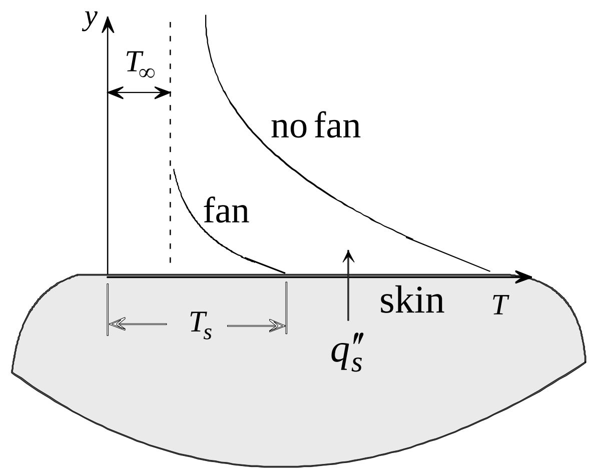
$$\left(\frac{\partial T}{\partial y}\right)_{y=0}(^{\text{o}}\text{C/m}) = -\frac{(q/A)_w(^{\text{o}}\text{C/m}^2)}{k(\text{W/m-}^{\text{o}}\text{C})} = {}^{\text{o}}\text{C/m}$$

Limiting check: (i) for $h = 0$ (no heat leaves the surface), surface temperature is infinite. Set $h = 0$ in (b) gives $T_s = \infty$.

(5) Comments. Temperature gradient at the surface is the same for all values of h as long as the thermal conductivity of the fluid is constant and radiation is neglected.

PROBLEM 1.4

Explain why fanning gives a cool sensation.



(1) Observations. (i) Metabolic heat leaves body at the skin by convection and radiation. (ii) Convection is described by Newton's law of cooling. (iii). Fanning increases the heat transfer coefficient and affects temperature distribution, including surface temperature. (iv). Surface temperature decreases as the heat transfer coefficient is increased. (v) Surface temperature is described by Newton's law of cooling. (vi) Ambient temperature is constant.

(2) Problem Definition. Determine effect of heat transfer coefficient on surface temperature.

(3) Solution Plan. Apply Newton's law of cooling to examine surface temperature.

(4) Plan Execution.

(i) Assumptions. (1) Steady state, (2) no radiation ,(3) uniform ambient fluid temperature, (4) constant surface heat flux and (5) constant properties.

(ii) Analysis. Newton's law of cooling

$$q_s'' = h(T_s - T_\infty) \tag{a}$$

where

h = heat transfer coefficient, $\text{W/m}^2-^\text{o}\text{C}$

q_s'' = surface heat flux, W/m^2

T_s = surface temperature, $^\text{o}\text{C}$

T_∞ =ambient temperature, $^\text{o}\text{C}$

Solve (a) for T_s

$$T_s = T_\infty + \frac{q_s''}{h} \tag{b}$$

This result shows that for constant q_s'', surface temperature decreases as h is increased. Since fanning increases h it follows that it lowers surface temperature and gives a cooling sensation.

(iii) Checking. *Dimensional check*: Each term in (b) has units of temperature

$$T_s(^\text{o}\text{C}) = T_\infty(^\text{o}\text{C}) + \frac{q_s''(\text{w/m}^2)}{h(\text{w/m}^2-^\text{o}\text{C})} = ^\text{o}\text{C}$$

Limiting check: for $h = 0$ (no heat leaves the surface), surface temperature is infinite. Set $h = 0$ in (b) gives $T_s = \infty$.

(5) Comments. (i) The analysis is based on the assumption that surface heat flux remains constant. (ii) Although surface temperature decreases with fanning, temperature gradient at the surface remains constant. This follows from the application of Fourier's law at the surface

$$q_s'' = -k\left(\frac{\partial T}{\partial y}\right)_s$$

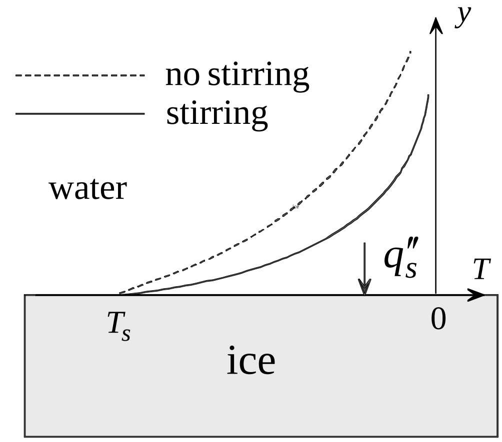
Solving for $(\partial T / \partial y)_s$

$$\left(\frac{\partial T}{\partial y}\right)_s = -\frac{q_s''}{k} = \text{constant}$$

PROBLEM 1.5

A block of ice is submerged in water at the melting temperature. Explain why stirring the water accelerates the melting rate.

(1) Observations. (i) Melting rate of ice depends on the rate of heat added at the surface. (ii) Heat is added to the ice from the water by convection. (iii) Newton's law of cooling is applicable. (iv). Stirring increases surface temperature gradient and the heat transfer coefficient. An increase in gradient or h increases the rate of heat transfer. (v) Surface temperature remains constant equal to the melting temperature of ice. (vi) water temperature is constant.



(2) Problem Definition. Determine effect of stirring on surface heat flux.

(3) Solution Plan. Apply Newton's law of cooling to examine surface heat flux.

(4) Plan Execution.

(i) Assumptions. (1) no radiation ,(2) uniform water temperature, (3) constant melting (surface) temperature.

(ii) Analysis. Newton's law of cooling

$$q_s'' = h(T_s - T_\infty) \tag{a}$$

where

h = heat transfer coefficient, $\mathrm{W/m^2-{}^oC}$

q_s'' = surface heat flux, $\mathrm{W/m^2}$

T_s = surface temperature, $^o\mathrm{C}$

T_∞ =ambient water temperature, $^o\mathrm{C}$

Stirring increases h . Thus, according to (a) surface heat flux increases with stirring. This will accelerate melting.

(iii) Checking. *Dimensional check*: Each term in (a) has units of heat flux.

Limiting check: For $T_\infty = T_s$ (water and ice are at the same temperature), no heat will be added to the ice. Set $T_\infty = T_s$ in (a) gives $q_s'' = 0$.

(5) Comments. An increase in h is a consequence of an increase in surface temperature gradient. Application of Fourier's law at the surface gives

$$q_s'' = -k\left(\frac{\partial T}{\partial y}\right)_s \tag{b}$$

PROBLEM 1.5 (continued)

Combining (a) and (b)

$$h = \frac{-k\left(\frac{\partial T}{\partial y}\right)_s}{T_s - T_\infty} \qquad \text{(c)}$$

According to (c), for constant T_s and T_∞, increasing surface temperature gradient increases *h*.

PROBLEM 1.6

Consider steady state, incompressible, axisymmetric parallel flow in a tube of radius r_o. The axial velocity distribution for this flow is given by

$$u = \bar{u}\,(1 - \frac{r^2}{r_o^2})$$

where $\bar{u}$ is the mean or average axial velocity. Determine the three components of the total acceleration for this flow.

(1) Observations. (i) This problem is described by cylindrical coordinates. (ii) For parallel streamlines $v_r = v_\theta = 0$. (iii) Axial velocity is independent of axial and angular distance.

(2) Problem Definition. Determine the total acceleration in the r, θ and z directions.

(3) Solution Plan. Apply total derivative in cylindrical coordinates.

(4) Plan Execution.

(ii) Assumptions. (1) Constant radius tube, (2) constant density and (3) streamlines are parallel to surface.

(ii) Analysis. Total acceleration in cylindrical coordinates is given by

$$\frac{dv_r}{dt} = \frac{Dv_r}{Dt} = v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} + \frac{\partial v_r}{\partial t} \tag{1.23a}$$

$$\frac{dv_\theta}{dt} = \frac{Dv_\theta}{Dt} = v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} + \frac{\partial v_\theta}{\partial t} \tag{1.23b}$$

$$\frac{dv_z}{dt} = \frac{Dv_z}{Dt} = v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial t} \tag{1.23c}$$

For streamlines parallel to surface

$$v_r = v_\theta = 0 \tag{a}$$

The axial velocity $u = v_z$ is given by

$$v_z = u = \bar{u}\,(1 - \frac{r^2}{r_o^2}) \tag{b}$$

From (b) it follows that

$$\frac{\partial v_z}{\partial z} = \frac{\partial v_z}{\partial t} 0 \tag{c}$$

Substituting into (1.23a), (1.23b) and (1.23c)

Radial acceleration: $$\frac{dv_r}{dt} = \frac{Dv_r}{Dt} = 0$$

Angular acceleration; $$\frac{dv_\theta}{dt} = \frac{Dv_\theta}{Dt} = 0$$

Axial acceleration: $$\frac{dv_z}{dt} = \frac{Dv_z}{Dt} = 0$$

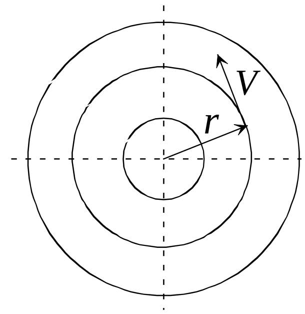
(5) Comments. All three acceleration components vanish for this flow.

PROBLEM 1.7

Consider transient flow in the neighborhood of a vortex line where the velocity is in the tangential direction given by

$$V(r,t) = \frac{\Gamma_o}{2\pi r}\left[1 - \exp\left(-\frac{r^2}{4\nu t}\right)\right]$$

Here r *is the radial coordinate,* t *is time,* Γ_o *is circulation (constant)* ν *is kinematic viscosity. Determine the three components of total acceleration.*



(1) Observations. (i) This problem is described by cylindrical coordinates. (ii) streamlines are concentric circles. Thus the velocity component in the radial direction vanishes ($v_r = 0$). (iii) For one-dimensional flow there is no motion in the *z*-direction ($v_z = 0$). (iv) The θ-velocity component, v_θ, depends on distance *r* and time *t*.

(2) Problem Definition. Determine the total acceleration in the *r*, θ and *z* directions.

(3) Solution Plan. Apply total derivative in cylindrical coordinates.

(4) Plan Execution.

(ii) Assumptions. (1) streamlines are concentric circles (2) no motion in the *z*-direction.

(ii) Analysis. Total acceleration in cylindrical coordinates is given by

The three components of the total acceleration in the cylindrical coordinates r, θ, z are

$$\frac{dv_r}{dt} = \frac{Dv_r}{Dt} = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z} + \frac{\partial v_r}{\partial t} \tag{1.23a}$$

$$\frac{dv_\theta}{dt} = \frac{Dv_\theta}{Dt} = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z} + \frac{\partial v_\theta}{\partial t} \tag{1.23b}$$

$$\frac{dv_z}{dt} = \frac{Dv_z}{Dt} = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial t} \tag{1.23c}$$

For the flow under consideration the three velocity component, v_r, v_θ and v_z are

$$v_r = 0 \tag{a}$$

$$v_\theta(r,t) = \frac{\Gamma_o}{2\pi r}\left[1 - \exp\left(-\frac{r^2}{4\nu t}\right)\right] \tag{b}$$

$$v_z = 0 \tag{c}$$

Radial acceleration: (a) and (c) into (1.23a)

$$\frac{Dv_r}{Dt} = -\frac{v_\theta^2}{r} \tag{d}$$

(b) into (d)

$$\frac{Dv_r}{Dt} = -\frac{(\Gamma_o)^2}{4\pi^2 r^3}\left[1 - \exp\left(-\frac{r^2}{4\nu t}\right)\right]^2 \tag{e}$$

Tangential acceleration: (a) and (c) into (1.23b)

$$\frac{Dv_\theta}{Dt} = \frac{\partial v_\theta}{\partial t} \tag{f}$$

(b) into (f)

$$\frac{Dv_\theta}{Dt} = -\frac{\Gamma_o}{2\pi r}\left(\frac{r^2}{4\nu t}\right)\frac{1}{t}\exp - \frac{r^2}{4\nu t} \tag{g}$$

Axial acceleration: (a) and into (1.23c)

$$\frac{Dv_z}{Dt} = 0 \tag{h}$$

(iii) Checking. *Dimensional check*: Units of acceleration in (e) and (g) are m/s^2. Note that according to (b), units of Γ_o are $\text{m}^2\text{/s}$ and the exponent of the exponential is dimensionless. Thus units of (e) are

$$\frac{Dv_r}{Dt} = -\frac{(\Gamma_o)^2(\text{m}^4/\text{s}^2)}{4\pi^2 r^3(\text{m}^3)}\left[1 - \exp\left(-\frac{r^2}{4\nu t}\right)\right]^2 = \text{m/s}^2$$

Units of (g) are

$$\frac{Dv_\theta}{Dt} = -\frac{\Gamma_o(\text{m}^2/\text{s})}{2\pi r(\text{m})}\left(\frac{r^2(\text{m}^2)}{4\nu(\text{m}^2/\text{s})t(\text{s})}\right)\frac{1}{t(\text{s})}\exp - \frac{r^2}{4\nu t} = \text{m/s}^2$$

Limiting check: (1) For $\Gamma_o = 0$, all acceleration components vanish. Setting $\Gamma_o = 0$ in (e) and (g) gives $\dfrac{Dv_r}{Dt} = \dfrac{Dv_\theta}{Dt} = 0$

(2) According to (b) at $t = \infty$, the tangential velocity vanishes ($v_\theta = 0$). Thus all acceleration components should vanish. Setting $t = \infty$ in (e) and (g) gives $\dfrac{Dv_r}{Dt} = \dfrac{Dv_\theta}{Dt} = 0$.

(5) Comments. The three velocity components must be known to determine the three acceleration components.

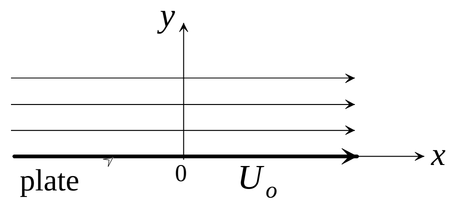
PROBLEM 1.8

An infinitely large plate is suddenly moved parallel to its surface with a velocity U_o. The resulting transient velocity distribution of the surrounding fluid is given by

$$u = U_o\left[1 - (2/\sqrt{\pi})\int_0^{\eta} \exp(-\eta^2)d\eta\right]$$

where the variable η is defined as

$$\eta(x,t) = \frac{y}{2\sqrt{\nu t}}$$



Here t is time, y is the vertical coordinate and ν is kinematic viscosity. Note that streamlines for this flow are parallel to the plate. Determine the three components of total acceleration.

(1) Observations. (i) This problem is described by Cartesian coordinates. (ii) For parallel streamlines the *y*-velocity component $v = 0$. (iii) For one-dimensional flow there is no motion in the *z*-direction (*w* = 0). The *x*-velocity component depends on distance *y* and time *t*.

(2) Problem Definition. Determine the total acceleration in the *x*, *y* and *z* directions.

(3) Solution Plan. Apply total derivative in Cartesian coordinates.

(4) Plan Execution.

(ii) Assumptions. (1) streamlines are parallel to surface and (2) no motion in the *z*-direction.

(ii) Analysis. Total acceleration in Cartesian coordinates is given by

$$\frac{df}{dt} = \frac{Df}{Dt} = u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} + \frac{\partial f}{\partial t} \tag{1.21}$$

where *f* represents any of the three velocity components *u*, *v* or *w*. The *x*-velocity component *u* is given by

$$u = U_o\left[1 - (2/\sqrt{\pi})\int_0^{\eta} \exp(-\eta^2)d\eta\right] \tag{a}$$

where

$$\eta(x,t) = \frac{y}{2\sqrt{\nu t}} \tag{b}$$

Note that *u* depends on *y* and *t* only. For one-dimensional parallel flow

$$v = w = 0 \tag{c}$$

Total acceleration in the *x*-direction, a_x. Set $f = u$ in (1.21)

$$a_x = \frac{du}{dt} = \frac{Du}{Dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \tag{d}$$

Since *u* depends on *y* and *t* only, it follows that

$$\frac{\partial u}{\partial x} = 0 \tag{e}$$

Substitute (c) and (e) into (d)

$$a_x = \frac{\partial u}{\partial t} \tag{f}$$

This derivative is obtained using the chain rule

$$a_x = \frac{\partial u}{\partial t} = \frac{du}{d\eta}\frac{\partial \eta}{\partial t} \tag{g}$$

Using (a)

$$\frac{du}{d\eta} = -\frac{2U_o}{\sqrt{\pi}}\exp(-\eta^2) \tag{h}$$

Using (b)

$$\frac{\partial \eta}{\partial t} = \frac{y}{2\sqrt{\upsilon}}t^{-3/2} = -\frac{y}{4\sqrt{\upsilon t}}\frac{1}{t} = -\frac{\eta}{4t} \tag{i}$$

Substitute (h) and (i) into (g)

$$a_x = \frac{\partial u}{\partial t} = \frac{U_o}{2\sqrt{\pi}}\frac{\eta \exp(-\eta^2)}{t} \tag{g}$$

Total acceleration in the *y*-direction, a_y. Set $f = v$ in (1.21)

$$a_y = \frac{dv}{dt} = \frac{Dv}{Dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \tag{h}$$

Apply (c) to (h)

$$a_y = 0 \tag{i}$$

Total acceleration in the *z*-direction, a_z. Set $f = w$ in (1.21)

$$a_w = \frac{dw}{dt} = \frac{Dw}{Dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \tag{j}$$

Apply (c) to (h)

$$a_z = 0 \tag{k}$$

(iii) Checking. *Dimensional check*: Units of acceleration in (g) are m^2/s. Note that η is dimensionless. Thus units of (g) are

$$a_x = \frac{U_o(\text{m/s})}{2\sqrt{\pi}}\frac{\eta \exp(-\eta^2)}{t(\text{s})} = \text{m}^2/\text{s}$$

Limiting check: (1) For $U_o = 0$, the acceleration $a_x = 0$. Setting $U_o = 0$ in (g) gives $a_x = 0$.

(2) According to (b) at $t = \infty$, $\eta(y,\infty) = 0$. Evaluation (a) at $\eta(y,\infty) = 0$ gives

PROBLEM 1.8 (continued)

$$u(y,\infty) = U_o \tag{l}$$

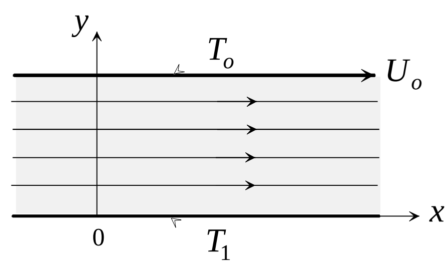
Since u is constant every where it follows that the a_x must be zero. Setting $\eta = 0$ and $t = \infty$ in (g) gives $a_x = 0$.

(5) Comments. The three velocity components must be known to determine the three acceleration components.

PROBLEM 1.9

Consider two parallel plates with the lower plate stationary and the upper plate moving with a velocity U_o. The lower plate is maintained at temperature T_1 and the upper plate at T_o. The axial velocity of the fluid for steady state and parallel streamlines is given by

$$u = U_o \frac{y}{H}$$



where H is the distance between the two plates. Temperature distribution is given by

$$T = \frac{\mu U_o^2}{2kH}\left[y - \frac{y^2}{H}\right] + (T_o - T_1)\frac{y}{H} + T_1$$

where k is thermal conductivity and μ is viscosity. Determine the total temperature derivative.

(1) Observations. (i) This problem is described by Cartesian coordinates. (ii) For parallel streamlines the *y*-velocity component $v = 0$. (iii) For one-dimensional flow there is no motion in the *z*-direction (*w* = 0). The *x*-velocity component depends on distance *y* only.

(2) Problem Definition. Determine the total temperature derivative.

(3) Solution Plan. Apply total derivative in Cartesian coordinates.

(4) Plan Execution.

(ii) Assumptions. (1) streamlines are parallel to surface, (2) no motion in the *z*-direction and (3) temperature distribution s one dimensional, $T = T(y)$.

(ii) Analysis. Total acceleration in Cartesian coordinates is given by

$$\frac{df}{dt} = \frac{Df}{Dt} = u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} + \frac{\partial f}{\partial t} \tag{1.21}$$

where *f* represents temperature. Let *f* = *T* in (1.21)

$$\frac{dT}{dt} = \frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} + \frac{\partial T}{\partial t} \tag{a}$$

where

$$u = U_o \frac{y}{H} \tag{b}$$

and

$$v = w = 0 \tag{c}$$

Temperature distribution is given by

$$T = \frac{\mu U_o^2}{2kH}\left[y - \frac{y^2}{H}\right] + (T_o - T_1)\frac{y}{H} + T_1 \tag{d}$$

Using (d)